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Superluminal effect in a hybrid nano-electro-optomechanical system

Aiman al Omari

Department of Physics, Jerash University, Jerash, Jordan

ABSTRACT

We theoretically investigate the phenomenon of superluminosity of the transmission probe field under the effect of a strong driving field in a hybrid nano-electro-optomechnical system comprising two coupled charged mechanical end resonators connected by bias voltages. The first mechanical charged mirror is connected to the optical field, while the second charged mirror is coupled by Coulomb coupling. We find that the superluminal behavior of the transmitted probe field can be switched by adjusting the cavity field detuning and atom field detuning. The present study extends the previously developed simple optomechanical and nano-electro-optomechanical system (Ma et al., 2014)[62]. Moreover, the group delay of the pulse advancement can be controlled by adjusting the electrostatic Coulomb coupling and the power of the driving field. We explain the effect of atomic field detuning on the superluminal behavior in the regimes of anti-Stokes sidebands, i.e. $\Delta_a = -\omega_1$.

1. Introduction

In recent years quantum optical systems with mechanical motion [1] has been one of the busiest research fields in which new techniques and methodologies are developing every day. The study of the subluminal and superluminal effects is one of them, a challenging phenomenon in this broad field. The superluminal effect of light was first observed by Chu and Wong in a resonant system Chu and Wong [2]. The theoretical aspect of slow and fast light was studied by Boyd and Gauthier [4], Bigelow et al. [5] and Wu and Deng [3] and experimentally was first observed by Kasapi et al. [6]. The slow and fast light in a carbon nanotube resonator with a two mode field system was studied by Li and Zhu [7], and recently in a double resonator system by Ma et al. [8]. The subluminal and superluminal properties of light in hybrid optomechanical systems have been discussed by Akram et al. [9], Safavi-Naeini et al. [10] and Li et al. [11].

The subluminal and superluminal phenomenon of light have many potential applications. To name a few, atomic vapors or Bose Einstein condensate and solid state media [12], all optical routing [13], slow and fast light in liquid crystal light valves [14], all optical switching [15], atomic frequency combs [16], reconfiguration of optical delay in a hot atomic vapors [17], quantum coherence phenomenon through a ruby crystal [18], photorefractive crystals through the strong dispersion of dynamic gratings in the vicinity of Bragg resonance [19], and photonics [20].

Different techniques are used to realize the subluminal and superluminal behavior of light, such as electromagnetically induced transparency (EIT) [21], coherence population oscillations (CPO) [22], stimulated Raman scattering (SRS) [23,24] and phonon induce transparency (PIT) [25–28], dispersion-compensating fibers (DCFs) [29], fiber Bragg gratings (FBGs) [30] and the implementation of tunable delay in coupled cavities [31]. Moreover, the effect of slow and fast in the shape of electromagnetically induced transparency in a linear coupled optomechanical system has been theoretically investigated by Sumei Huang and Agarwal [32] and experimentally demonstrated by Weis et al. [33].

A hybrid optomechanical system consists of a nano-mechanical resonator (mechanical membranes) with other mechanical resonators [34–37], ultra cold atoms [38,39] are applied for different dynamics. Due to the unique properties of optomechanics, it has

E-mail address: aiman_101@yahoo.com.

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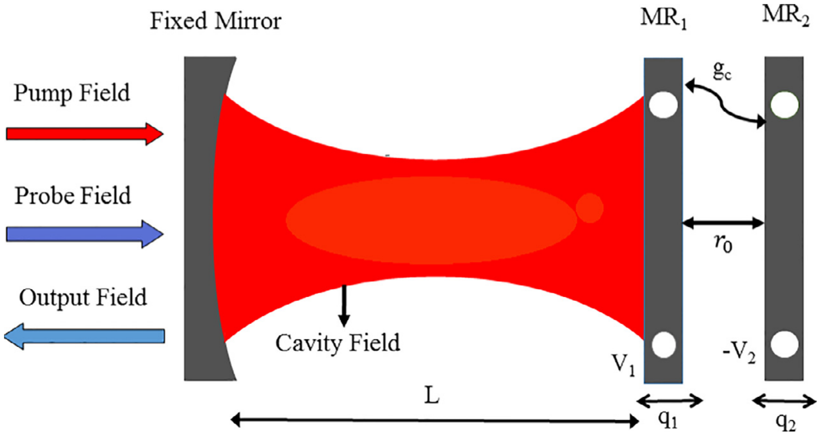

Fig. 1. Schematic demonstration of the nano-electro-optomechanical system: MR_1 is coupled to the cavity field as well as to the second mechanical resonator (MR_2) through the Coulomb coupling g_c. MR_1 is charged by the bias voltage V_1, while MR_2 is charged by the bias voltage $-V_2$. The cavity mode is driven by a strong input laser field of amplitude E_l and a weak probe field of amplitude ϵ_p trough the fixed mirror. Where L is the dimension of the cavity length. σ_z is the transition operator between the two levels. r_0 is the equilibrium distance between MR_1 and MR_2.

many features which can be demonstrated in particular cases. For example, the normal mode splitting (NMS) in optomechanics is observed, whenever a strong driven field is sent inside an optomechanical system, it raises the radiation pressure and the coupling of the system becomes sensitive and splits the modes [40–46]. Other applications of the optomechanical system with radiation pressure are the Fano resonances [47], squeezing [48], quantum entanglement [49–54], normal mode splitting [55], and mechanically induced transparency [56]. On the basis of these advancements, he hybrid optomechanical systems provide a bridge for quantum electrodynamics (QED) devices [57–61] and quantum nano-electro-optomechanical systems (NEOMS) [8,62] in which charged resonators are coupled through Coulomb coupling. Based on these works, it can be seen that additional media, like a two level atom play an important role in NEOMS. But the subluminal and superluminal behavior of light have been rarely studied in this particular hybrid system. Therefore, we can expect good results for the subluminal and superluminal effects of light by studying such hybrid quantum nano-electro-optomechanical systems. In this paper, we consider an optomechanical system coupled with a mechanical resonator through a Coulomb force. In our system (i) two field frequencies are controlled by a single driving field. (ii) The system is robust against the cavity decay. (iii) In our robust system, we provide a controllable fast light by controlling the system parameters.

This paper is divided into the following sections: In Section 2, we present the model and discuss its theoretical background. In Section 3, we follow the quantum Langevin's approach and derive the analytical results of the system. In Section 4, we discuss the numerical results. In Section 4, we present our concluding remarks.

2. Theory and mathematical formalism of NEOMS

We consider a hybrid nano-electro-optomechanical system, which consists of an optomechanical system where the optical field is coupled with a resonator that interacts with another mechanical resonator. The charge on the first resonator (MR_1) is $q_1 = V_1C_1$, while the charge on the second resonator (MR_2) is $q_2 = -V_2C_2$ due to the external bias voltages V_1 and $-V_2$, as shown in Fig. 1. Here, C_1 (C_2) is the capacitance due to the external bias voltage V_1 ($-V_2$). Thus, MR_1 is not only coupled to the cavity field but also coupled with MR_2, so that it shares the optomechanical coupling g_0 with the cavity field and the Coulomb coupling g_c with MR_2. The Coulomb coupling can be switched on or off by the bias voltages V_1 and $-V_2$. The Hamiltonian of the whole system can be written as

$$\begin{aligned} H \;=\; & \hbar\omega_c\hat{c}^\dagger\hat{c} + \left(\frac{\hat{p}_1^{\,2}}{2m_1} + \frac{1}{2}m_1\omega_1^2\hat{q}_1^{\,2}\right) + \left(\frac{\hat{p}_2^{\,2}}{2m_2} + \frac{1}{2}m_2\omega_2^2\hat{q}_2^{\,2}\right) - \hbar g_0\hat{c}^\dagger\hat{c}\hat{q}_1 + \hbar g_c\hat{q}_1\hat{q}_2 \\ & + i\hbar E_l\left(\hat{c}^\dagger e^{-i\omega_l t} - H.c\right) + i\hbar\left(\hat{c}^\dagger\epsilon_p e^{-i\omega_p t} - H.c\right), \end{aligned} \tag{1}$$

where the first term is for the single-mode cavity field, the second term and third term represent the oscillations of MR_1 and MR_2 corresponding to the effective masses m_1 and m_2. The fourth term represents the coupling between the cavity field and MR_1 given by $g_0 = \frac{\omega_c}{L}\sqrt{\frac{\hbar}{m_1\omega_1}}$. The fifth term comes from the Coulomb coupling between MR_1 and MR_2 and is related to $g_c = \frac{-q_1q_2}{2\pi\hbar\epsilon_0 r_0^3}$ [63]. Here, r_0 is the equilibrium distance between MR_1 and MR_2. The last two terms represent the input driving field and the probe field with amplitudes E_l and ϵ_p, where $E_l = \sqrt{\frac{2\wp_l\kappa}{\hbar\omega_l}}$ and $\epsilon_p = \sqrt{\frac{2\wp_p\kappa}{\hbar\omega_p}}$, $\wp_l$ and $\wp_p$ are the laser and probe field powers. Moreover, $\hat{c}$ and $\hat{c}^\dagger$ are the annihilation and creation operators of the cavity field, $\hat{q}_1(\hat{q}_2)$ and $\hat{p}_1(\hat{p}_2)$ are the position and momentum operators of MR_1 and MR_2. In the rotating frame corresponding to the laser frequency ω_l the Hamiltonian of the system becomes

$$\begin{aligned} H &= \hbar\Delta_c \hat{c}^\dagger \hat{c} + \left(\frac{\hat{p}_1^2}{2m_1} + \frac{1}{2}m_1\omega_1^2\hat{q}_1^2\right) + \left(\frac{\hat{p}_2^2}{2m_2} + \frac{1}{2}m_2\omega_2^2\hat{q}_2^2\right) \\ &\quad - \hbar g_0 c^\dagger \hat{c}\hat{q}_1 + \hbar g_c \hat{q}_1\hat{q}_2 + i\hbar E_l(\hat{c}^\dagger - \hat{c}) + i\hbar(\hat{c}^\dagger \epsilon_p e^{-i\delta t} - \hat{c}\epsilon_p^\star e^{i\delta t}), \end{aligned} \tag{2}$$

where $\Delta_c = \omega_c - \omega_l$ and $\delta = \omega_p - \omega_l$ are the cavity field and probe field detuning corresponding to the laser frequency. In order to discuss the solutions of the system, we consider the decay terms γ_1, γ_2 for MR_1 and MR_2. The mean values equations of the NEOMS with the mean field approximation $\langle \hat{c}^\dagger \hat{c}\rangle = \langle \hat{c}^\dagger\rangle\langle \hat{c}\rangle$ [32] can be written as

$$\begin{aligned} \langle \dot{\hat{q}}_1\rangle &= \frac{\langle \hat{p}_1\rangle}{m_1}, \\ \langle \dot{\hat{q}}_2\rangle &= \frac{\langle \hat{p}_2\rangle}{m_2}, \\ \langle \dot{\hat{p}}_1\rangle &= -m_1\omega_1^2\langle \hat{q}_1\rangle + g_0\langle \hat{c}^\dagger\rangle\langle \hat{c}\rangle - g_c\langle \hat{q}_2\rangle - \gamma_1\langle \hat{p}_1\rangle, \\ \langle \dot{\hat{p}}_2\rangle &= -m_2\omega_2^2\langle \hat{q}_2\rangle - g_c\langle \hat{q}_1\rangle - \gamma_2\langle \hat{p}_2\rangle, \\ \langle \dot{\hat{c}}\rangle &= -(\kappa + i\Delta_c)\langle \hat{c}\rangle + ig_0\langle \hat{c}\rangle\langle \hat{q}_1\rangle. \end{aligned} \tag{3}$$

Here $\hbar = 1$. The Brownian noise terms, which are zero in the mean field approximation, have been dropped. Eq. (3) is the nonlinear set of equations of the steady state. To find out the steady-state values of the NEOMS, we take the steady state operator [12,32], $\langle \hat{O}\rangle = \hat{O}_s + \hat{O}_+\epsilon_p e^{-i\delta t} + \hat{O}_-\epsilon_p^\star e^{i\delta t}$ such that $\hat{O} = \hat{O}_s + \delta\hat{O}$ and $\hat{O}_s >> \delta\hat{O}$. Here, $\hat{O}_s$ represents the steady state value of $\hat{c}_s$, $\hat{q}_{is}$, $\hat{p}_{is}$, $i = 1, 2$, and the $\hat{O}_\pm$ are treated as perturbations. Keeping the lowest order terms of the steady-state mean value solutions, we get

$$\begin{aligned} \hat{p}_{1s} &= \hat{p}_{2s} = 0, \\ \hat{q}_{1s} &= \frac{g_0|c_s|^2}{m_1\omega_1^2 - \frac{g_c^2}{m_2\omega_2^2}}, \\ \hat{q}_{2s} &= \frac{-g_0 g_c|c_s|^2}{m_2\omega_2^2(m_1\omega_1^2 - \frac{g_c^2}{m_2\omega_2^2})}, \\ \hat{c}_s &= \frac{E_l}{\kappa + i\Delta}, \\ \Delta &= \Delta_c - g_0\hat{q}_{1s}, \end{aligned} \tag{4}$$

$$c_- = \epsilon_p[A - B + C - B^2(A' + B + C')^{-1}]^{-1}, \tag{5}$$

where

$$\begin{aligned} A &= \kappa + i(\Delta - \delta), \\ B &= \frac{G_s}{\alpha - \beta}, C = \frac{ig_a^2}{\delta} = C', \\ \alpha &= m_1(\omega_1^2 - \delta^2 - i\delta\gamma_1), \\ \beta &= \frac{g_c^2}{m_2(\omega_2^2 - \delta^2 - i\delta\gamma_2)}, \\ G_s &= g_0^2|c_s|^2, \\ A' &= \kappa - i(\Delta + \delta). \end{aligned}$$

In addition, at detuning $\delta_c = \omega_{1,2}$ and $\Delta = \delta$, the coupling between the cavity field and MR_1 becomes stronger. For simplification, we consider that the system is in the sideband resolved regime, i.e., $\omega_{1,\,2} >> \kappa$ and $\delta_c \sim \omega_{1,\,2}$ [32]. In order to investigate the optical property of the out field, we recall the input and output standard relation of the cavity field [64].

$$\begin{aligned} 2\kappa\langle \hat{c}\rangle &= \sqrt{2\kappa}\langle \hat{c}_{out}\rangle + \hat{c}_{in}, \\ 2\kappa\langle \hat{c}\rangle &= \sqrt{2\kappa}\langle \hat{c}_{out}\rangle + \epsilon_p e^{-i\delta t} + E_l. \end{aligned} \tag{6}$$

Here [66] we define a new variable for the output field such that $\epsilon_{out} = \sqrt{2\kappa}\langle \hat{c}_{out}\rangle + \epsilon_p e^{-i\delta t} + E_l$, therefore $\epsilon_{out} = 2\kappa\langle c\rangle$, described as

$$\epsilon_{out} = \epsilon_{out0} + \epsilon_{out+}\epsilon_p e^{-i\delta t} + \epsilon_{out-}\epsilon_p^\star e^{i\delta t}, \tag{7}$$

where ϵ_{out0}, ϵ_{out+} and ϵ_{out-} are the components of the output field corresponding to the frequencies ω_c, ω_p, $2\omega_c - \omega_p$. Here ϵ_{out+} represents the nonlinear effect and generates the four wave mixing, while ϵ_{out-} is called anti-Stokes and is responsible for electromagnetically induced transparency (EIT). Now we write the rescaled output probe field frequency by inserting Eq. (7) in Eq. (6), we get

$$\begin{aligned} \epsilon_{out+} &= \frac{2\kappa c_+}{\epsilon_p}, \\ \epsilon_{out-} &= \frac{2\kappa c_-}{\epsilon_p}. \end{aligned} \tag{8}$$

The output field amplitude ϵ_{out} appears as a complex number, which can be resolved numerically into real and imaginary parts. Both ϵ_{out+} and ϵ_{out-} represent the reflection part of the output probe field. The transmission part of the output probe field can be expressed in the form of

$$t_p(\omega_p) = 1 - \frac{2\kappa c_\pm}{\epsilon_p}. \tag{9}$$

The output probe field accounts for the in-phase and out-phase corresponding to the quadratures $Re(\epsilon_{out}) = \frac{\kappa(c_- + c_-^\star)}{\epsilon_p}$ and $Im(\epsilon_{out}) = \frac{\kappa(c_- - c_-^\star)}{i\epsilon_p}$, which describe the absorption and dispersion of the system with respect to the probe field. In the nano-electro-optomechanical system the driven field frequency not only brings changes in the output probe field but also brings rapid changes in the phase of the transmission field. The phase dispersion relation of the transmitted output probe field is given by

$$\phi_t(\omega_p) = arg[\epsilon_{out}]. \tag{10}$$

This expression can also be written as

$$\phi_t(\omega_p) = \arctan\left[\frac{Im(\epsilon_{out})}{Re(\epsilon_{out})}\right]. \tag{11}$$

The phase causes the transmission group delay [65], which can be written as

$$\tau_g = \frac{d\phi_t(\omega_p)}{d(\omega_p)}. \tag{12}$$

The negative group delay $\tau_g < 0$ corresponds to fast light. We support our analytical work with a simulation based on the realizable experimental parametric values for fast light, we choose the realizable parameters values to demonstrate the slow and fast light effect based on the atom-assisted nano-electro-optomechanical system. Here all the used parameters are accessible in experiment [33,36,41,42,44]: length of the cavity $L = 25$ nm, $\omega_{1,2} = 2\pi \times 947$ kHz, $\kappa = 2\pi \times 215$ kHz, $Q = 6700$, $\gamma_{1,2} = \omega_{1,2}/Q$, $m_1 = m_2 = 145$ ng, $\delta = \omega_{1,2}$, the wavelength of the driving field $\lambda_l = 1064$ nm, the optomechanical coupling $g_0 = 2\pi \times 8$ kHz, and the Coulomb coupling $g_c = 2\pi \times 100$ kHz. The cavity field detuning $\Delta_c = 2\pi \times 40$ kHz.

3. Superluminal effect of light

In this section, we demonstrate the superluminal effect of NEOMS. In Fig. 2(a–f), we show the normalized plot of the transmission field in term of the probe field with amplitude ϵ_p for different values of the optomechanical coupling. In Fig. 2(a) a Lorentzian line shape was made to appear by switching off the optomechanical coupling, i.e., $g_0 = 0$ [9]. Once the optomechanical coupling is introduced the Lorentzian line shape (single EIT window) splits into a double EIT window, as shown in Fig. 2(b). The peak height of the transparency window at the middle on either side (belongs to optomechanical coupling) gradually increases by increasing the optomechanical coupling. The height becomes prominent for higher values of g_0, as labeled in Fig. 2(c–f). The dip of the window represents the superluminal effect, while the peak represents the subluminal effect of light. The physical significance of Fig. 2(a–f) follows the phenomenon of detuning, $\delta = \omega_{1,2}$ corresponds to $\delta = \omega_p - \omega_l$, which generates the radiation pressure by which the frequency of the driven field is shifted from the Stokes to the anti-Stokes regime $\omega_l - \omega_m$. This phenomenon is caused by the probe field frequency. Moreover, the frequency of the probe field and anti-Stokes field generates the destructive interferences, as a result the probe field can suppress the build-up of an intracavity field; this leads to a narrow transmission window, as shown in Fig. 2(a–f). In addition, the width of the spacing directly depends on the power of the driven field. The same effect can be observed by increasing the Coulomb coupling through the bias voltages.

In Fig. 3(a–c), we plot the rapid phase dispersion relation versus normalized detuning δ/ω_1 of the transmission probe field. In Fig. 3(a), all the three couplings are switched off, no dip is shown, which means there is no group delay of the transmitted output probe field. However, when the optomechanical coupling comes into account one dip appears, as shown in Fig. 3(b); as a result we have one group delay of the transmitted probe field. Moreover, by the addition of coulomb coupling the phase dispersion has two dips around $\delta = \omega_1$, as a result two group delay has appeared in Fig. 3(c). The curve in Fig. 3(b,c) represents the anomalous dispersion of the transmitted field with a significant high dispersion in the presence of Coulomb coupling. The physical significance of the phase dispersion relation is a manifestation of the slow and fast light of the transmitted output probe field. Thus the higher the phase dispersion relation the greater the change in the group delay index of the medium.

In Fig. 4(a,b), we plot the group delay τ_g of the phase dispersion as a function of δ/ω_1 corresponding to $\wp_l = 4\mu W$, $\delta = \omega_1$, and $\Delta_a = \omega_1$ for different values of the cavity decay rate κ and Coulomb coupling g_c. For experimental realizable parameters, we have pulse advancement of $\tau_g = 1.5$ μs for $\kappa = 2\pi \times 215$ kHz and $\tau_g = 15$ μs for $g_c = 2\pi \times 100$ kHz (black solid curves) in Fig. 4(a,b). The next pulse advancement of group delay become $\tau_g = 0.85$ μs, 27.30 μs for $\kappa = 3\pi \times 215$ kHz and $g_c = 2\pi \times 200$ kHz (cyan dashed

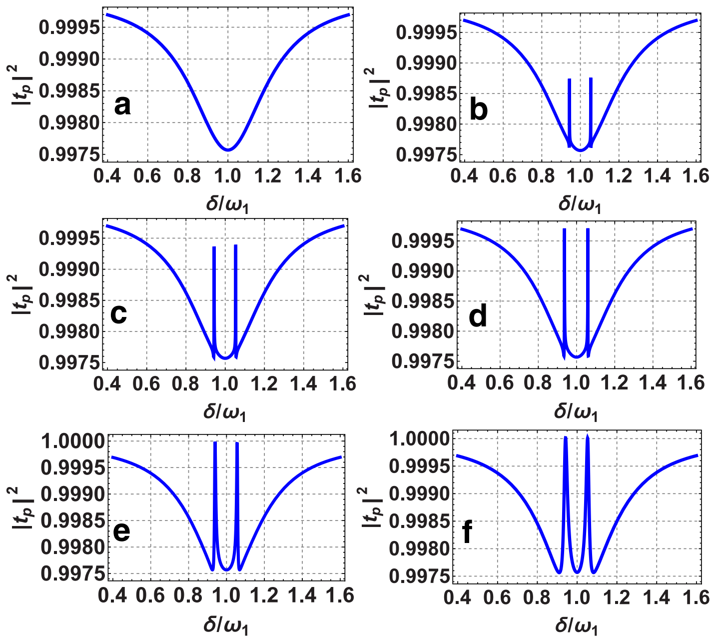

Fig. 2. We sketch the transmission output probe field $|t_p|^2$ as a function of normalized detuning δ/ω_1. (a) $g_0 = 0$. (b) $g_0 = 2\pi \times 0.5$ kHz. (c) $g_0 = 2\pi$ kHz. (d) $g_0 = 2\pi \times 1.5$ kHz. (e) $g_0 = 2\pi \times 2$ kHz. (f) $g_0 = 2\pi \times 2.5$ kHz. Other parameters are $\kappa = 2\pi \times 215$ kHz, cavity field detuning $\Delta = \omega_1$, $m_{1,2} = 145$ ng and Coulomb coupling $g_c = 2\pi \times 100$ kHz.

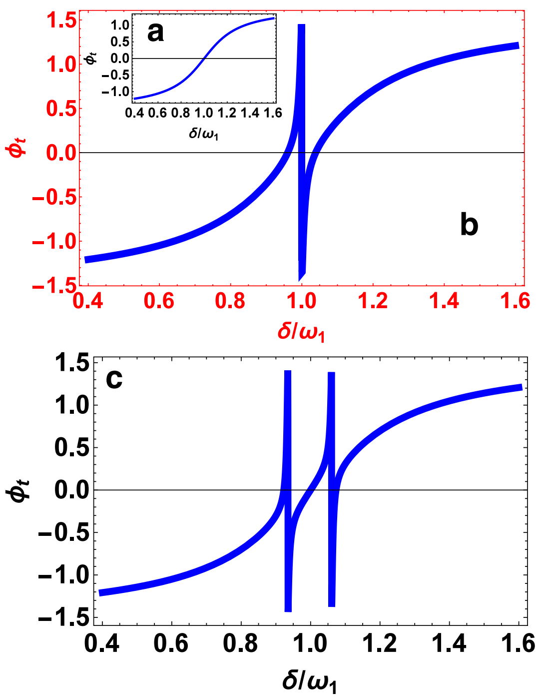

Fig. 3. We show the phase dispersion of the transmission output probe field ϕ_t as a function of the normalized detuning δ/ω_1 for $g_{ac}\sqrt{N} = 0$. (a) $g_0 = 0$, $g_c = 0$. (b) $g_0 = 2\pi \times 4$ kHz and $g_c = 0$. (c) $g_0 = 2\pi \times 4$ kHz and $g_c = 2\pi \times 100$ kHz. Other parameters are the same as used in Fig. 2.

curves). Hence, the response of the optical field to the weak probe field leads to a group delay. We see in Fig. 4(a,b) that the group delay is negative for the output probe field for $\delta = \omega_1$, which shows the fast light effect of the transmitted field. In Fig. 4(b) the group delay causes the rapid change in the group velocity of the transmitted probe field due to suffering the sharpness of the phase dispersion. It is clear from Fig. 4(a–b), that upon increasing the value of κ the magnitude of the advance pulse decreases. Hence, the greater the group delay the more obvious it is to see the fast light effect.

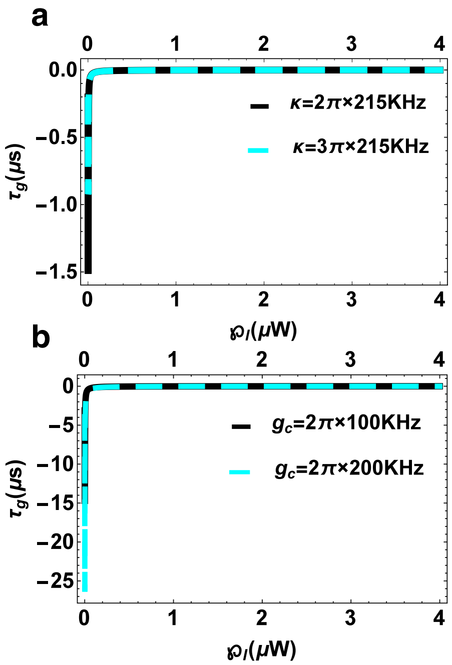

Fig. 4. We plot the group delay τ_g as a function of pump power $\wp_l$ for the case of $\Delta_a = \omega_{1,2}$ (a) $\kappa = 2\pi \times 215$ kHz (black solid curve) and $\kappa = 3\pi \times 215$ kHz (cyan dashed curve). (b) $g_c = 2\pi \times 100$ kHz (black solid curve) and $g_c = 2\pi \times 200$ kHz (cyan dashed curve). Other parameters are $g_0 = 2\pi \times 4$ kHz, $\gamma_{1,2} = 2\pi \times 140$ kHz and $\delta = \omega_1$.

4. Conclusion

We discussed the subluminal and superluminal effects of light in a hybrid nano-electro-optomechanical system which consists of two coupled charged resonators (MR_1 and MR_2) and a fixed mirror. We derived the analytical expression for the transmission field and phase dispersion relations corresponding to group delays on the basis of input and output theory. The phase dispersion indicates two group delays of the transmitted field. The main advantage of our hybrid system is the fact that it can be operated at room temperature. Our hybrid NEOMS model provides a new way for some aspiring potential applications in the present day technology of optics and photonics, consisting of integrated quantum optomechanical memory, real quality imaging, designing novel quantum information processing gates, higher detection efficiency of X-rays, optical buffering, delay lines, and telecommunication.

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