# Variation of Quality Factors and Bandwidth of A Conically Depressed Microstrip Patch Antenna in Plasma Medium 

Ayman Al-Sawalha<br>Physics Department, Faculty of Science, Jerash University, Jerash 31982, Jordan<br>* E-mail of the corresponding author: dr_sawalha@hotmail.com


#### Abstract

In the present paper theoretical investigation has been carried out to evaluate the effect of plasma wave on the radiation properties of a conically depressed microstrip patch antenna .Using linearized hydrodynamic theory and potential function technique, the total directive gain and quality factor are calculated for different values of plasma to source frequency $\left(\omega_{p} / \omega\right)$ and different half-cone angles. It is observed that the directive gain and quality factor are changes considerably by changing the half-cone angle $\psi$ as well as a plasma to source frequency $\left(\omega_{p} / \omega\right)$ values.


Keywords: Microstrip Antenna, Plasma Medium, Radiated power, Directive gain, Quality factor.

## 1. Introduction

Printed circuit antennas have been applied to a variety of systems such as high-flying airplanes and satellites [1] due to their light weight, low cost and easy fabrication technique. However it is low bandwidth becomes a disadvantages and hence reduce it is application for different purposes. Constant efforts are going on to improve the
bandwidth of such radiators. One way to improve the bandwidth is that the patch surface of a circular microstrip antenna is depressed slightly into the substrate material which gives the structure of a conically depressed microstrip antenna [2]. Another way is, by increasing the substrate thickness between the ground plane and the conducting patch [3]. Radiation properties of a conically depressed radiator are investigated in a homogeneous warm electron plasma of infinite extent. The presence of the plasma excites an electron plasma (EP) waves in addition to the usual electromagnetic (EM) waves. Radiation properties of a circular patch microstrip antenna are already studied in plasma medium using cavity model and results are presented elsewhere [4].Due to the conical depression of circular patch surface into the substrate by an angle $\psi$, an additional electric field in the radial direction will be introduced which will give rise to extra radiations. Using this additional field in addition to the already existing fields, radiation properties of a conically depressed microstrip patch antenna are studied in warm ionized plasma medium.

## 2. Formulation of the Problem and Basic Equations

The geometry and coordinates system of a dimensionally thin conically depressed microstrip patch antenna is shown in figure (1).


Figure (1): The geometry and coordinates system of a conically depressed microstrip patch antenna.

A circular patch $\left(\psi=90^{\circ}\right)$ of radius $a$ in (xy) plane is depressed conically $\left(\psi=\psi^{o}\right)$ along the $z$-axis. Thickness of the substrate is considered to be $h$, relative permittivity and permeability are $\varepsilon_{r}>1$ and $\mu_{r}=1$ respectively. Basic assumptions and initial equations regarding a plasma medium are given in [5]. The internal fields in region $R_{1}$ of such a radiator, excited in $T M$ mode are:

$$
\begin{equation*}
E_{z}=E_{o} J_{n}(k \rho) \cos (n \varphi) \tag{1}
\end{equation*}
$$

From Maxwell's equations, we obtain

$$
\begin{align*}
& H_{\rho}=-j \frac{\omega \varepsilon n}{k^{2} \rho} E_{o} J_{n}(k \rho) \sin (n \varphi)  \tag{2}\\
& H_{\varphi}=-j \frac{\omega \varepsilon}{k} E_{o} J_{n}^{\prime}(k \rho) \cos (n \varphi) \tag{3}
\end{align*}
$$

Where $n$ is an integer, $k=\omega\left(\mu_{o} \varepsilon_{o} \varepsilon_{r}\right)^{1 / 2}, \varepsilon_{o}$ is the free space permittivity, $\varepsilon_{r}$ is the relative permittivity of substrate, $\varepsilon=\varepsilon_{r} \varepsilon_{o}, J_{n}$ is the Bessel function of order $n$ and $J_{n}^{\prime}(k \rho)$ is the derivative of $J_{n}(k \rho)$ with respect to the argument $(k \rho)$. Due to the depression of a conducting circular patch into the dielectric substrate by an angle $\psi$, the internal fields will be modified. In region $R_{2}$, a radial field $E_{\rho}$ will be existed. The magnitude of $E_{\rho}$ is zero at $z=0$ and equal to $E_{z} \cot \psi$ at $z=\rho \cot \psi$ ( as a boundary condition) with uniform variation along the $z$ direction. Excitation of $E_{\rho}$ in region $R_{2}$ will create $H_{z}$ and $E_{\varphi}$ in addition to the field components already existing in
region $R_{1}$ In region $R_{2}$ the $E_{z}$ and $E_{\rho}$ components are related as:

$$
\begin{equation*}
E_{z}=(-\rho / z) E_{\rho} \tag{4}
\end{equation*}
$$

And $E_{\rho}$ at resonance will be given by:

$$
\begin{equation*}
E_{\rho}=\left(-\frac{\omega \mu_{o}}{k^{2} \rho}\right)\left(\frac{\partial H_{z}}{\partial \varphi}\right) \tag{5}
\end{equation*}
$$

Where,

$$
\begin{equation*}
H_{z}=\left(\frac{-j \omega \varepsilon \rho}{n}\right) \cot \psi E_{o} J_{n}(k \rho) \sin n \varphi \tag{6}
\end{equation*}
$$

Following the method of [6], the far zone components of electromagnetic mode and plasma mode are computed. These are :

### 2.1. In Electromagnetic Mode:

$E_{\theta}=j^{n} \frac{V_{o} \beta_{e} a_{e}}{2 r} e^{-j \beta_{e} r} \cos (n \varphi) \frac{\sin \left(0.5 \beta_{e} h \cos \theta\right)}{0.5 \beta_{e} h \cos \theta}\left[J_{n+1}\left(a_{e} \beta_{e} \sin \theta\right)-J_{n-1}\left(a_{e} \beta_{e} \sin \theta\right)\right]\left[1+\frac{a}{h} \cot \psi\right]$
$E_{\varphi}=j^{n} \frac{V_{o} \beta_{e} a_{e}}{2 r} e^{-j \beta_{e} r} \cos \theta \sin (n \varphi) \frac{\sin \left(0.5 \beta_{e} h \cos \theta\right)}{0.5 \beta_{e} h \cos \theta}\left[J_{n+1}\left(a_{e} \beta_{e} \sin \theta\right)-J_{n-1}\left(a_{e} \beta_{e} \sin \theta\right)\right]\left[1+\frac{a}{h} \cot \psi\right]$ here
$V_{o}=h E_{o} J_{n}(k a)$ is the edge voltage at $\varphi=0$, and $\beta_{e}=\left(\frac{2 \pi}{\lambda_{o}}\right) A$ is the propagation constant in electromagnetic mode.

### 2.2. In Plasma Mode:

$$
\begin{equation*}
E_{p}=(-j)^{n+2} \frac{\beta_{p} h E_{o} n \omega_{p}^{2}}{r \omega\left(\omega^{2}-\omega_{p}\right)} e^{-j \beta_{p} r} J_{n}(k a) \sin (n \varphi) J_{n}\left(\beta_{p} a_{e} \sin \theta\right) \frac{\sin \left(0.5 \beta_{p} h \cos \theta\right)}{0.5 \beta_{p} h \cos \theta}\left[1+\frac{a}{h} \cot \psi\right] \tag{9}
\end{equation*}
$$

Where $\beta_{p}=\left(\frac{c}{\vartheta_{o}}\right) \beta_{o} A$ is the propagation constant in plasma mode and $\beta_{o}$ is the propagation constant in free space.

## 3-Radiated Power

The total power radiated by an antenna in the plasma medium is a sum of the power radiated in the electromagnetic mode $P_{e}$ and in the plasma mode $P_{p}$ as mentioned in [7]. The radiated power in:

### 3.1. Electromagnetic Mode

The power radiated in the electromagnetic mode in the upper half space is obtained by integrating the complex Poynting vector [8]. Thus, the power radiated in the electromagnetic mode can be expressed as:

$$
\begin{equation*}
P_{e}=\frac{A}{2 Z_{0}} \int_{0}^{2 \pi} \int_{0}^{\pi}\left[\left|E_{\theta}\right|^{2}+\left|E_{\varphi}\right|^{2}\right] r^{2} \sin \theta d \theta d \varphi \tag{10}
\end{equation*}
$$

Substituting the values of $E_{\theta}$ and $E_{\varphi}$ from equations (7) and (8) and performing the integral for the $\varphi$ variables, the total radiated power can be expressed as:

$$
\begin{equation*}
P_{e}=\frac{A \pi}{4} \sqrt{\frac{\varepsilon_{o}}{\mu_{o}}} V_{o}^{2} \frac{\sin ^{2}\left(0.5 h \beta_{e} \cos \theta\right)}{\left(0.5 h \beta_{e} \cos \theta\right)^{2}}\left[\beta_{e}^{2} a_{e}^{2} I_{1}+I_{2}\right]\left[1+\frac{4 \varepsilon_{e f f}}{A^{2}}\left(a \cot \psi^{2}\right)\left(\frac{a^{2}}{h^{2}}\right)\right] \tag{11}
\end{equation*}
$$

where

$$
\begin{align*}
& I_{1}=\int_{0}^{\pi} J_{1}^{\prime 2}\left(\beta_{e} a_{e} \sin \theta\right) \sin \theta d \theta  \tag{12}\\
& I_{2}=\int_{0}^{\pi} J_{1}^{2}\left(\beta_{e} a_{e} \sin \theta\right) \frac{\cos ^{2} \theta}{\sin \theta} d \theta \tag{13}
\end{align*}
$$

The integrals in the above two equations have been evaluated numerically.

### 3.2. Plasma Mode

The radiated power of an antenna in the plasma mode is given as

$$
\begin{equation*}
P_{p}=\frac{A}{1-A^{2}}\left(\frac{v_{o}}{c}\right) \frac{1}{2 Z_{o}} \int_{0}^{2 \pi} \int_{0}^{\pi}\left|E_{p}\right|^{2} r^{2} \sin \theta d \theta d \varphi \tag{14}
\end{equation*}
$$

Where $Z_{o}$ is the impedance in the free space.
Substituting equation (9) into equation (14), the radiated power in the plasma mode can be expressed as:

$$
\begin{equation*}
P_{p}=\frac{30 \pi^{2}\left(1-A^{2}\right)}{A}\left(\frac{c}{v_{o}}\right) V_{o}^{2} \int_{0}^{\pi}\left[\frac{\sin \left(0.5 h \beta_{p} \cos \theta\right)}{\left(0.5 h \beta_{p} \cos \theta\right)} J_{n}\left(\beta_{p} a_{e} \sin \theta\right)\right]^{2}\left[1+\left(\frac{a^{2}}{h^{2}}\right)^{2} \cot ^{2} \psi\right] \sin \theta d \theta \tag{15}
\end{equation*}
$$

## 4. Quality Factor

A term specifying the frequency selectivity of the resonant circuit is the quality factor. It is defined as the ratio between the energy stored and the energy lost in the system. At resonance, the energy stored can be calculated either from the maximum magnetic field or from the maximum electric field. In case of a conically depressed microstrip antenna, the total energy stored $\left(U_{T}\right)$ is defined as $U_{T}=U_{1}+U_{2}+U_{3}$, where $U_{1}$ is the energy due to $E_{z}$ in region $R_{1}, U_{2}$ is the energy due to $E_{z}$ in region $R_{2}$ and $U_{3}$ is the energy due to $E_{\rho}$ in region $R_{2}$. In general the energy stored is given by,

$$
\begin{equation*}
U=\frac{\varepsilon_{o} \varepsilon_{e f f}}{4} \int_{v}\left|E_{\max }\right|^{2} d v \tag{16}
\end{equation*}
$$

Therefore,

$$
\begin{align*}
& U_{1}=\frac{\pi E_{o}^{2}(h-a \cot \psi)}{4 \omega^{2} \mu_{o}}\left(\beta_{o}^{2} a^{2}-1\right) J_{1}^{2}(k a)  \tag{17}\\
& U_{2}=\frac{2 \pi \beta_{o}^{2} E_{o}^{2} \cot \psi}{4 \omega^{2} \mu_{o}}\left[a^{2} J_{2}^{2}(k a)\right] \tag{18}
\end{align*}
$$

$$
\begin{equation*}
U_{2}=\frac{2 \pi \beta_{o}^{2} E_{o}^{2} \cot ^{3} \psi}{16 \omega^{2} \mu_{o}}\left[a^{2} J_{2}^{2}(k a)\right] \tag{19}
\end{equation*}
$$

Thus, the total radiation loss factor $\left(Q_{R}\right)$ will given by :

$$
\begin{equation*}
Q_{R}=\frac{\omega U_{T}}{P_{r}} \tag{20}
\end{equation*}
$$

A quality factor has been calculated for different values of plasma parameter $A$ and half-cone angle $\psi$, in both the modes as shown in table (1).

Table (1): Quality factor of a conically patch microstrip antenna for different values of plasma parameter $A$ and half-cone angle $\psi$.

| $A$ | $\psi=90^{\circ}$ |  | $\psi=85^{\circ}$ |  | $\psi=75^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q_{e} \times 10^{3}$ | $Q_{p} \times 10^{3}$ | $Q_{e}$ | $Q_{p} \times 10^{3}$ | $Q_{e}$ | $Q_{p} \times 10^{3}$ |
| 1.0 | 0.163 | - | 55.330 | - | 6.870 | - |
| 0.9 | 0.208 | 394.570 | 57.280 | 0.310 | 7.100 | 3.848 |
| 0.8 | 0.279 | 70.897 | 60.530 | 0.615 | 7.52 | 0.077 |
| 0.7 | 0.394 | 36.720 | 65.510 | 0.030 | 8.140 | 0.356 |
| 0.6 | 0.595 | 6.395 | 72.700 | 0.490 | 9.030 | 0.062 |
| 0.5 | 0.993 | 6.673 | 84.170 | 0.520 | 10.400 | 0.064 |
| 0.4 | 1.867 | 6.554 | 101.310 | 0.510 | 21.600 | 0.063 |
| 0.3 | 4.278 | 3.110 | 130.470 | 0.250 | 16.350 | 0.032 |
| 0.2 | 14.130 | 1.817 | 191.700 | 0.150 | 23.820 | 0.017 |
| 0.1 | 93.150 | 1.208 | 315.910 | 0.094 | 39.200 | 0.117 |

## 5. Bandwidth

Bandwidth of an antenna mainly depend on its impedance and patterns. From the resonant circuit theory, bandwidth can be defined by the following relation,

$$
\begin{equation*}
B W=\omega_{2}-\omega_{1}=\Delta \omega=\frac{\omega_{r}}{Q} \tag{21}
\end{equation*}
$$

Where, $\omega_{r}$ is the resonant frequency. Using the relation (20), the bandwidth of a conically depressed microstrip antenna is calculated, the variation of bandwidth with plasma parameter (A) for different half-cone angle $(\psi)$ are given in table(2).

Table (2): Bandwidth of a conically patch microstrip antenna for different values of plasma parameter $A$ and half-cone angle $\psi$.

| $A$ | $\psi=90^{\circ}$ | $\psi=85^{\circ}$ | $\psi=75^{\circ}$ |
| :---: | :---: | :---: | :---: |
|  | BW\% | BW\% | BW\% |
| 1.0 | 10.4 | 31.0 | ------- |
| 0.9 | 8.2 | 24.4 | 196.2 |
| 0.8 | 6.3 | 19.9 | 160.8 |
| 0.7 | 4.4 | 13.1 | 105.9 |
| 0.6 | 3.2 | 9.8 | 78.6 |
| 0.5 | 2.0 | 5.9 | 47.8 |
| 0.4 | 1.2 | 3.3 | 26.2 |
| 0.3 | 0.9 | 1.8 | 14.6 |
| 0.2 | 1.0 | 1.9 | 15.2 |
| 0.1 | 1.4 | 2.1 | 17.1 |

## 6. Result and Discussion

When an antenna interacts with the plasma medium, in addition to the usual electromagnetic waves, electroacoustic waves also be generated. Presence of these electroacoustic waves perhaps changes the radiation properties of such a radiator in plasma medium. Presence of plasma medium effects the bandwidth of the antenna. For all structures, bandwidth is maximum in free space but decreases on decreasing plasma parameter value (A).

In free space the quality factor $(Q)$ is maximum for the structure having $\left(\psi=90^{\circ}\right)$ and it increases on reducing plasma parameter value (A). From the study of various parameters, it can be concluded that the present geometry provides some useful results both in free space as well as in the plasma medium. The magnitude of fields and directivity of such antenna decreases sharply, but radiated power, bandwidth increases considerably as half-cone angles $(\psi)$ decreases. This make such an antenna a useful structure for communication purpose.

## 7. References

1. J.R.Wait , "Radiation from sources immersed in compressible plasma media",Can.J.Physics,vol.42, 1964.
2. C.A. Balanis, " Antenna Theory: Analysis and Design",2nd ed.,John Wiley and Sons,New York, 2005.
3. S.Yoon,R.W.Ziolkowski," Bandwidth of a microstrip patch antenna on a magneto
dielectric substrate", IEEE ,Soc.Int.Symposium,Columbus,pp.297-300,2003.
4. H. Griguer, E. Marzolf, H. Lalj, F. Riouch and M. Drissi."em Patch antenna Bandwidth Enhancement through the use of metamaterials", International conference on telecommunications. pp. 323-327.2009.
5. A.G.Derneyd," New technology for microwave antennas",Workshop on Antennas.19th European Microwave Conference,Germany,1989.
6. K.R.Carver "Practical analysis techniques for the microstrip antenna",Workshop on printed antenna,New Mexico State University, 1998.
7. I. J. Bahl and P.Bhartia," Microstrip Antennas", Artech House, Massachusetts, 1995.
8. D.Bhatnagar,R.K.Gupta," On radiation from microstrip patch in two components warm plasma medium",Indian J.of Physics,Vol.68B,pp.92-96,1992.

This document was created with Win2PDF available at http://www.win2pdf.com.
The unregistered version of Win2PDF is for evaluation or non-commercial use only.

